Lecturered By Dr. H. K. Jadav
Deptt. of Mathematics Marwari College, D'bga
B.Sc. part- II (Sub), date - 01-05-2020

Differential calculus. (maxima and Minima)

Q.1. Find the maximum and minimum values of
$2x^3 - 21x^2 + 36x - 20$

Soln: Let $y = 2x^3 - 21x^2 + 36x - 20$ ——— (I)

Differentiate w.r.t. x, we get

$\frac{dy}{dx} = 6x^2 - 42x + 36$ ——— (II)

and $\frac{d^2y}{dx^2} = 12x - 42$ ——— (III)

For maximum or minimum values of y, we have

$\frac{dy}{dx} = 0$.

$\Rightarrow 6x^2 - 42x + 36 = 0 \Rightarrow x^2 - 7x + 6 = 0$

$\Rightarrow x^2 - 6x - x + 6 = 0 \Rightarrow x(x-6) - 1(x-6) = 0$

$\Rightarrow (x-1)(x-6) = 0$, $\therefore x = 1$ or $x = 6$

When $x = 1$, then from III, $\frac{d^2y}{dx^2} = -30 < 0$.

Hence, y is maximum, when $x = 1$

From (I), the maximum value of $y = -3$

When $x = 6$, then from III, $\frac{d^2y}{dx^2} = 30 > 0$

Hence, y is minimum, when $x = 6$.

From (I). the minimum value of y

$= 2 \times 6^3 - 21 \times 6^2 + 36 \times 6 - 20$

$= 432 - 756 + 216 - 20 = -128$.

[signature]

Q.9. Find the maximum value of $x^{\frac{1}{x}}$.

Soln: Let $y = x^{1/x}$

Taking logarithm in both sides, we get

$$\log y = \frac{1}{x}\log x$$

Differentiating w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = -\frac{1}{x^2}\log x + \frac{1}{x^2} = \frac{1}{x^2}(1-\log x) \quad \text{— (I)}$$

For maximum or minimum of y, $1 = \log x \Rightarrow x = e$

Differentiating (I) w.r.t. x, we get

$$-\frac{1}{y^2}\left(\frac{dy}{dx}\right)^2 + \frac{1}{y}\frac{d^2y}{dx^2} = \frac{-2}{x^3}(1-\log x) - \frac{1}{x^3}$$

when $x = e$, $\frac{1}{y}\cdot\frac{d^2y}{dx^2} = -\frac{1}{e^3}$

$$\therefore \frac{d^2y}{dx^2} = -e^{\frac{1}{e}}\cdot\frac{1}{e^3} < 0.$$

$\therefore$ y is maximum when $x = e$ and the maximum value of $= e^{\frac{1}{e}}$.